Bounds On Anomalous Magnetic And Electric Moments
Of Tau Lepton From LEP And Lepton Flavor Violation

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ABSTRACT

The most stringent bounds on the anomalous magnetic and electric dipole moments of the tau lepton was derived by Escribano and Massó[6] from the Z width at LEP in an effective lagrangian approach to the new physics. In this paper we point out that the higher dimensional operators introduced by Escribano and Massó not only modify the neutral currents of the tau lepton to Z gauge boson and the photon, but also induce lepton flavor violation (LFV). The size of the LFV effect depends crucially on the dynamics of the lepton mass generation. Assuming the lepton mass matrices in the form of an Fritzsch ansatz, we point out that the experimental limit on $\mu \to e\gamma$ will push the anomalous magnetic and electric dipole moments of the tau lepton down to 10^{-11} and 10^{-25} e cm respectively.

PACS: 13.10.+q, 12.60.Cn

Although the Standard Model (SM) has been successful in describing the physics of the electroweak interaction[1], it is quite possible that the SM is only an effective theory which breaks down at higher energies as the structure of the underlying physics emerges. There are reasons to believe that the deviation from the SM might first appear in the interactions involving the third-generation fermions[2]. In the lepton sector, the tau lepton is special in probing new physics since it is the only lepton which is heavy enough to have hadronic decays and it is generally believed that the heavier fermions are more sensitive to the new physics related to mass generation than light fermions. There has been extensive studies on the tau physics in the literature[3]. In particular, there has been growing interest in the electromagnetic and electric dipole moments of the tau lepton[4][5] in recent years. In general, the anomalous magnetic and the electric dipole moments of the tau lepton are defined by (which we follow the notation of Ref.[6])

$$a_\tau = \frac{g_\tau - 2}{2} = F_2(q^2 = 0), \text{ and } d_\tau = e\tilde{F}_2(q^2 = 0), \tag{1}$$

where F_2 and $\tilde{F}_2$ are form factors in the electromagnetic matrix element

$$< p'|J^\mu_{em}(0)|p> = e\bar{u}(p')(F_1\gamma^\mu + [\frac{i}{2m_\tau}F_2 + \gamma_5\tilde{F}_2]\sigma^{\mu\nu}q_\nu)u(p), \tag{2}$$

where $q = p' - p$ and $F_1(q^2 = 0) = 1$.

Theoretically the standard model predicts $a_\tau = 1.1769(4) \times 10^{-3}$ and a very tiny d_τ[4] from CP violation in the quark sector. Experimentally recent analysis of the $e^+e^- \to \tau^+\tau^-\gamma$ process from L3 and OPAL collaborations give that $a_\tau = 0.004 \pm 0.027 \pm 0.023$ and $d_\tau = (0.0 \pm 1.5 \pm 1.3) \times 10^{-16} e$ cm[5]. However the most stringent bounds are that inferred from the width $\Gamma(Z \to \tau^+\tau^-)$ [6], which are $-0.004 \le a_\tau \le 0.006$ and $|d_\tau| \le 1.1 \times 10^{-17} e$ cm. To obtain these limits, Escribano and Massó took an effective lagrangian approach to new physics,

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \frac{1}{\Lambda^2}\sum_i C_i\mathcal{O}_i, \tag{3}$$

where $\mathcal{L}_0$ is the SM Lagrangian, Λ is the new physics scale and O_i are $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant operators. C_i are constants which represent the coupling strengths of O_i. A complete list of CP-violating and CP-conserving operators have been given in Ref.[7]. Regarding the

anomalous magnetic moment of the tau lepton, the authors of Ref.[6] have examined two dimension-six operators

$$\mathcal{O}_{\tau B} = \bar{L}\sigma^{\mu\nu}\tau_R\Phi B_{\mu\nu}, \tag{4}$$

$$\mathcal{O}_{\tau W} = \bar{L}\sigma^{\mu\nu}\vec{\sigma}\tau_R\Phi\vec{W}_{\mu\nu}, \tag{5}$$

where $L = (\nu_\tau, \tau_L)$, Φ is the Higgs scalar, $B_{\mu\nu}$ and $W_{\mu\nu}$ are field strengths of $U_Y(1)$ and $SU_L(2)$.

When Φ gets vacuum expectation value, operators $\mathcal{O}_{\tau B}$ and $\mathcal{O}_{\tau W}$ give rise to anomalous magnetic moment of the tau lepton and also corrections to the decay width of Z into $\bar{\tau}\tau$. Given that the experimental data on Z width is quite consistent with the prediction of the SM, Escribano and Massó put a strong bound on anomalous magnetic moment of the tau lepton listed above. In this paper, we extend the work of Ref.[6] by allowing the mixing of the three generations and point out that the experimental limits on LFV will put stronger bounds than Ref.[6] on anomalous magnetic and electric dipole moments of the tau lepton.

We present our arguments first with anomalous magnetic moment of the tau lepton. Considering the dimension-six operators, $\mathcal{O}_{\tau B}$ and $\mathcal{O}_{\tau W}$, the full effective lagrangian, $\mathcal{L}_{eff}$ now can be written as:

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \frac{1}{\Lambda^2}(c_{\tau B}\mathcal{O}_{\tau B} + c_{\tau W}\mathcal{O}_{\tau W} + h.c.). \tag{6}$$

After the electroweak symmetry is broken and the mass matrices of the fermions and the gauge bosons are diagonalized, the effective neutral current couplings of the leptons to gauge boson Z and the photon γ are

$$\mathcal{L}_{eff}^{Z,\gamma} = eg^{Z,\gamma}\begin{pmatrix}\bar{e}\\\bar{\mu}\\\bar{\tau}\end{pmatrix}^T\left\{(\gamma_\mu V^{Z,\gamma} - \gamma_\mu\gamma_5 A^{Z,\gamma}) - \frac{1}{2m_\tau}(ik_\nu\sigma^{\mu\nu})S^{Z,\gamma}U_l\begin{pmatrix}0 & & \\ & 0 & \\ & & 1\end{pmatrix}U_l^\dagger\right\}\begin{pmatrix}e\\\mu\\\tau\end{pmatrix}, \tag{7}$$

where $g^Z = 1/(4s_Wc_W)$, $g^\gamma = 1$, and $V^{Z,\gamma} = 1-4s_W^2, 1$, $A^{Z,\gamma} = 1, 0$ for Z and photon respectively, and

$$S^Z = -\frac{8s_Wc_W}{e}\frac{m_\tau}{\Lambda^2}\frac{v}{\sqrt{2}}\left[C_{\tau W}c_W - 2C_{\tau B}s_W\right], \tag{8}$$

$$S^\gamma = \frac{2m_\tau}{e}\frac{\sqrt{2}v}{\Lambda^2}\left[C_{\tau W}\frac{s_W}{2} - C_{\tau B}c_W\right]. \tag{9}$$

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The matrix U_l in Eq.(7) is the unitary matrix which diagonalizes the mass matrix of the charged lepton. In the SM, which corresponds to $\mathcal{L}_{eff}$ in the limit of $\Lambda \to \infty$, the matrix U_l is not measurable because of the zero neutrino masses and furthermore, the universality of the gauge interaction guarantees the absence of the flavor changing neutral current, the lepton flavor violation in the lepton sector.

The relative size of the $Z(\gamma)\tau\bar{\tau}$ to the flavor changing couplings $Z(\gamma)\tau\bar{\mu}$, $etc.$, in Eq.(7) depends on the rotation matrix U_l. The elements of U_l can be evaluated once the corresponding mass matrix is given. An interesting ansatz in the literature is the one suggested by Fritzsch and its variations [8-12]. The latter in Ref.[9] is given by

$$U_l = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \tag{10}$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$ and the three mixing angles are determined by corresponding lepton masses,

$$\tan\theta_{12} = -1 + \frac{2}{\sqrt{3}}\sqrt{\frac{m_e}{m_\mu}}, \tag{11}$$

$$\tan\theta_{23} = -\sqrt{2} - \frac{3}{\sqrt{2}}\frac{m_\mu}{m_\tau}, \tag{12}$$

$$\tan\theta_{13} = -\frac{2}{\sqrt{6}}\sqrt{\frac{m_e}{m_\mu}}. \tag{13}$$

With U_l in Eqs.(10-13), we have

$$V_l = U_l \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} U_l^\dagger = \begin{pmatrix} s_{13}^2 & s_{13}s_{23}c_{13} & s_{13}c_{23}c_{13} \\ s_{13}s_{23}c_{13} & s_{23}^2c_{13}^2 & s_{23}c_{13}^2c_{23} \\ s_{13}c_{23}c_{13} & s_{23}c_{13}^2c_{23} & c_{23}^2c_{13}^2 \end{pmatrix}$$
$$= \begin{pmatrix} 0.0035 & -0.049 & 0.032 \\ -0.049 & 0.701 & -0.455 \\ 0.032 & -0.455 & 0.296 \end{pmatrix}. \tag{14}$$

In the numerical calculation, we take that $\sqrt{m_e/m_\mu} \approx 0.0696$, and $m_\mu/m_\tau \approx 0.0594$.

The decay width of $l \to l' + \gamma$ is given by

$$\Gamma_{(l \to l'\gamma)} = \frac{m_l}{32\pi}\left(V_{ll'}eS^\gamma \frac{m_l}{m_\tau}\right)^2, \tag{15}$$

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where $V_{ll'}$ $(l \neq l')$ are the nondiagonal elements of matrix V defined in Eq. (14). In Eq. (15), We have neglected the mass of the light lepton l'.

Given the current experimental upper limits on $\mu^- \to e^- \gamma$, 4.9×10^{-11} [13], we have

$$|S^\gamma| < 1.3 \times 10^{-10}. \tag{16}$$

From Eqs.(7-9), the new physics contribution to the anomalous magnetic moments of the tau lepton is given by

$$|\delta\alpha_\tau| = |V_{\tau\tau}S^\gamma|. \tag{17}$$

With the bounds on S^γ in (16) and $V_{\tau\tau}$ in (14), we obtain that

$$|\delta\alpha_\tau| \leq 3.9 \times 10^{-11}. \tag{18}$$

This limit is much stronger than that in Ref.[6]. The limits from other LFV processes, such as $\tau^- \to e^- \gamma$, are weaker than that in Eq. (18).

Similarly, considering operators below which are introduced in Ref.[6],

$$\tilde{\mathcal{O}}_{\tau B} = \bar{L}\sigma^{\mu\nu}i\gamma_5\tau_R\Phi B_{\mu\nu}, \tag{19}$$

$$\tilde{\mathcal{O}}_{\tau W} = \bar{L}\sigma^{\mu\nu}i\gamma_5\vec{\sigma}\tau_R\Phi\vec{W}_{\mu\nu}, \tag{20}$$

and following the procedure above in obtaining the bounds on tau lepton magnetic moment, we put limit on the anomalous electric dipole moment

$$|d_\tau| \leq 2.2 \times 10^{-25} \ e \text{ cm}. \tag{21}$$

Again this is stronger than that obtained by Escribano and Massó.

In summary, we extend the work by Escribano and Massó to bound the anomalous magnetic and electric dipole moments of the tau lepton in the effective lagrangian by allowing the mixing of three generations in the lepton sector. In the standard model, these mixing effects are not measurable because of vanishing neutrino masses and the universal gauge interactions. With non-universal interaction, the lepton flavor violation happens even with zero neutrino

masses. By taking the lepton mass matrix of Fritzsch ansatz, we have demonstrated that the experimental limit on $\mu \to e\gamma$ put stronger limits on the anomalous magnetic and electric dipole moments of the tau lepton than obtained by Escribano and Massó which is listed in the particle data book[13]. However our results depend on the lepton mass matrix. So future experimental data on anomalous magnetic and electric dipole moments of the tau lepton together with lepton flavor violation will provide an experimental test on various lepton mass ansatz.

Acknowledgments

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